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Prolegomena to Conditional Expected Utility Maximiser's Preference Logic

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Abstract

As von Wright ([31]) points out, the study of preference logic faces the problem that almost every principle proposed as fundamental to one preference logic may be rejected by another one. Mullen ([18]) observes that this problem arises from the mistaken belief that concept construction of preference can satisfactorily be carried out in isolation from theory construction. In this paper we propose a new version of preference logic (CEUMPL) based on conditional expected utility maximisation theory, which can furnish a solution to this problem. Conditional expected utility maximisation can be a valid decision rule for decision makings under certainty, risk, uncertainty and ignorance. It has the merit of covering such a wide scope. We provide CEUMPL with a Domotor-type ([5]) semantics that is measurement-theoretic. From a measurement-theoretic viewpoint of decision theory, there is a tradition to explain an agent's beliefs and desires in terms of his preferences [and vice versa]. This explanation takes the form of a representation theorem of conditional expected utility maximisation: if [and only if] an agent's preferences satisfy such-and-such conditions, there exist a probability function and a utility function such that he should act as a conditional expected utility maximiser. The "if" part of each representation theorem of conditional expected utility maximisation can provide the measurability conditions of an agent's preferences for his beliefs and desires. Domotor's representation theorem is the only known one of conditional expected utility maximisation that has the "only if" part. So only by virtue of Domotor's representation theorem, we can explain an agent's preferences in terms of his beliefs and desires via conditional expected utility maximisation. We provide CEUMPL with a model by developing the idea of Naumov ([19]) and provide CEUMPL with a proof system by developing that of Segerberg ([25]). The semantics of Packard's ([20]) preference logic is also based on conditional expected utility maximisation. But this logic is incomplete. CEUMPL, on the other hand, has the merit of being not only complete but also decidable.

Key Words

preference logic, measurement theory, representation theorem, decision theory, projective geometry, separation theorem.

1 Introduction

The notion of preference plays an important role in many disciplines, including philosophy and economics.[1] Some of notable recent developments in ethics make substantial use of preference logic.[2] In computer science, preference logic has become an indispensable device. The founder of preference logic is the founding father of logic itself, Aristotle. Book III of the *Topics* can be regarded as the first treatment of the subject. From the 1950s to the 1960s, the study of preference logic flourished in Scandinavia—particularly by Halldén ([6]) and von Wright ([30]) and in the U.S.A.—particularly by Martin ([17]) and Chisholm and Sosa ([4]). Recently with the help of Boutilier's idea ([3]), van Benthem, Otterloo and Roy reduced preference logic to multi-modal logic ([29]).

Mullen refers to the following problem the study of preference logic faces:

> As von Wright ([31]) has recently commented, the development of a satisfactory logic of preference has turned out to be unexpectedly problematic. The evidence for this lies in the fact that almost every principle which has been proposed as fundamental to a preference logic has been rejected by some other source. [[18]: 247]

For example, the status of such logical properties as (transitivity), (contraposition), (conjunctive expansion), (disjunctive distribution) and (conjunctive distribution) is as follows:

Example 1.

	von Wright ([30])	Martin	Chisholm and Sosa
Transitivity	+	+	+
Contraposition	−	+	−
Conjunctive Expansion	+	−	−
Disjunctive Distribution	−	−	−
Conjunctive Distribution	+	−	−

'+' denotes the property in question being provable in the logic in question. '−' denotes the property in question not being provable in the logic in question. (Conjunctive expansion) says that an agent does not prefer φ_1 to φ_2 iff he does not prefer $\varphi_1 \wedge \neg\varphi_2$ to $\varphi_2 \wedge \neg\varphi_1$. (Disjunctive distribution) says that if he does not prefer $\varphi_1 \vee \varphi_2$ to φ_3, then he does not prefer φ_1 to φ_3 or does not prefer φ_2 to φ_3. (Conjunctive distribution) says that if he does not prefer φ_1 to φ_2 and does not prefer φ_3 to φ_2, then he does not prefer $\varphi_1 \vee \varphi_3$ to φ_2.

Mullen observes upon the cause of the above problem as follows:

> The mistake, therefore, upon which the philosophical logic of preference rests is the belief that concept construction [of preference] can satisfactorily be carried out in isolation from theory construction. The most general conclusion is then that preference logician

1 [8] gives a comprehensive survey of preference in general.

2 [7] gives a comprehensive survey of preference logic.

must also be a value theoretician, since a logic of preference can only be tested in the process of testing the more general theory in which it is embedded. [[18]: 255]

The aim of this paper is to propose a new version of complete and decidable preference logic—conditional expected utility maximiser's preference logic (CEUMPL) by virtue of which we can explain an agent's preferences in terms of his beliefs and desires via conditional expected utility maximisation theory, which can furnish a solution to this problem. We provide CEUMPL with a Domotor-type ([5]) semantics that is a kind of measurement-theoretic and decision-theoretic one. The semantics of Packard's ([20]) preference logic is also based on conditional expected utility maximisation. But this logic is incomplete. CEUMPL, on the other hand, has the merit of being not only complete but also decidable.

On the basis of [[15]: 13] with a slight modification, we can classify decision problems into the following four types. We say that an agent is in the realm of decision making under:

1. *Certainty* if each leads to a specific outcome with the probability of 1 that is known to him,
2. *Risk* if each action leads to one of a set of possible specific outcomes each of which occurs with a probability that is known to him,
3. *Uncertainty* if each action leads to one of a set of possible specific outcomes, some of which occur with a probability that is known to him, but the other of which occur with a probability that is unknown to him,
4. *Ignorance* if each action leads to one of a set of possible specific outcomes each of which occurs with a probability that is unknown to him.

Conditional expected utility maximisation can be a valid decision rule for decision makings under certainty, risk, uncertainty and ignorance. It has the merit of covering such a wide scope.

Measurement theory is one that can provide measurement with its mathematical foundation.[3] The mathematical foundation of measurement had not been studied before Hölder developed his axiomatisation for the measurement of mass ([9]). [14], [26] and [16] are seen as milestones in the history of measurement theory. In measurement theory, at least four kinds of measurement have been objects of study:

1. ordinal measurement,
2. extensive measurement,
3. difference measurement,
4. conjoint measurement.

On the other hand, there are at least two kinds of decision theory:

3 [23] gives a comprehensive survey of measurement theory.

1. evidential decision theory,[4]
2. causal decision theory.[5]

The former is designed for decision makings that have statistical or evidential connections between actions and outcomes. The latter is designed for decision makings that have causal connections between actions and outcomes. Both theories take the form of subjective expected utility theory. Jeffrey ([11]) is a typical example of the former. Ramsey ([21]) is a typical example of the latter. Ramsey regarded desire as attitude toward outcomes but belief as one toward propositions. Moreover, he regarded preference as attitude toward an ordered pair of gambles, that is, hybrid entities composed of outcomes and propositions. In 1965 Jeffrey ([11]) developed an alternative to Ramsey's theory. He regarded both desire and belief as attitudes toward propositions. Moreover, he regarded preference as attitude toward an ordered pair of propositions. In this sense we call Jeffrey's a mono-set theory. Its initial axiomatisation was provided in terms of measurement theory by Bolker ([2]) on the mathematics developed in [1]. Jeffrey ([10]) modified Bolker's axioms to accommodate null propositions. Domotor ([5]) also axiomatised a version of mono-set theory. Mono-set theories are more suitable for the semantics of logic than Ramsey's, for regarding propositions as the semantic values of sentences is simpler than regarding gambles as those when we wish to provide logic with its semantics. Especially, Domotor's theory is the most suitable for the semantics of logic of these three mono-set theories, for constructing the syntactic counterparts of the axioms of Domotor's theory is easier than of the other two theories.

Like Bolker's and Jeffrey's, Domotor's theory has a conjoint structure. In them, preferences are decomposable into beliefs and desires. From a measurement-theoretic viewpoint of decision theory, there is a tradition to explain an agent's beliefs and desires in terms of his preferences [and vice versa]. This explanation takes the form of a representation theorem of [conditional] expected utility maximisation:

> If [and only if] an agent's preferences satisfy such-and-such conditions, there exist a probability function and a utility function such that he should act as a conditional expected utility maximiser (*existence*). [In addition, the pair of such probability function and utility function is unique up to a kind of transformation (*uniqueness*).]

The "if" part of each representation theorem of conditional expected utility maximisation can provide the measurability conditions of an agent's preferences for his beliefs and desires. Domotor's representation theorem is the only known one of conditional expected utility maximisation that has the "only if" part. So only by virtue of Domotor's representation theorem, we can explain an agent's preferences in terms of his beliefs and desires via conditional expected utility maximisation.

The structure of this paper is as follows. In Section 2, we prepare the projective-geometric concepts for the measurement-theoretic settings: characteristic function, Grassmann product,

4 [11] gives a comprehensive survey of evidential decision theory.

5 [13] gives a comprehensive survey of causal decision theory.

symmetric product and four-fold Grassmann product, and define preference space and preference space assignment, and state necessary and sufficient conditions for representation: (connectedness) and (projectivity), and prove a representation theorem. In Section 3, we define the language $\mathcal{L}_{\mathsf{CEUMPL}}$ of CEUMPL, and define a Domotor-type structured Kripke model $\mathcal{M}$ for preference, and provide CEUMPL with a truth definition, and provide CEUMPL with a proof system, and show that (reflexivity), (transitivity), (connectedness) and (impartiality) are all provable in CEUMPL, and that neither (contraposition), (conjunctive expansion), (disjunctive distribution) nor (conjunctive distribution) is provable in CEUMPL, but that (restricted contraposition), (restricted conjunctive expansion), (restricted disjunctive distribution) and (restricted conjunctive distribution) are all provable in CEUMPL, and prove the soundness, completeness and decidability of CEUMPL.

2 Measurement-Theoretic Settings

2.1 Projective-Geometric Concepts

We need some projective-geometric concepts to state Domotor's representation theorem. We define the preliminaries to the measurement-theoretic settings as follows:

Definition 1 (Preliminaries). $\mathbf{W}$ *is a nonempty set of possible worlds. Let* $\mathcal{F}$ *denote a Boolean field of subsets of* $\mathbf{W}$. *We call* $A \in \mathcal{F}$ *a proposition.*

We define a characteristic function as follows:

Definition 2 (Characteristic Function I). *A characteristic function* $\widehat{\ } : \mathcal{F} \to \{0,1\}^{\mathbf{W}}$ *is one where for any* $A \in \mathcal{F}$ we have $\widehat{A} : \mathbf{W} \to \{0,1\}$ *such that*

$$\widehat{A}(w) := \begin{cases} 1 & \text{if } w \in A, \\ 0 & \text{otherwise}, \end{cases}$$

for any $w \in \mathbf{W}$.

Because it is impossible to characterise multiplication of probabilities and utilities in terms of union, intersection and preferences, we need a Cartesian product $\times$. $\widehat{\ }$ is defined also on Cartesian products of propositions:

Definition 3 (Characteristic Function II).

$$(A \times B)\widehat{\ }(w_1, w_2) := \begin{cases} 1 & \text{if } w_1 \in A \text{ and } w_2 \in B, \\ 0 & \text{otherwise}, \end{cases}$$

for any $w_1, w_2 \in \mathbf{W}$.

By means of $\times$, we define a Grassmann product $\widehat{A} \circ \widehat{B}$ as follows:

Definition 4 (Grassmann Product). *$\widehat{A} \circ \widehat{B}$ is a 3-valued random variable defined by*

$$\widehat{A} \circ \widehat{B} := (A \times B)\widehat{\ } - (B \times A)\widehat{\ }.$$

By means of $\circ$, we define a symmetric product $\odot(\widehat{A},\widehat{B},\widehat{C},\widehat{D})$ as follows:

Definition 5 (Symmetric Product). *$\odot(\widehat{A},\widehat{B},\widehat{C},\widehat{D})$ is a 9-valued random variable defined by*

$$\begin{aligned}
&\odot(\widehat{A},\widehat{B},\widehat{C},\widehat{D})\\
&:= (\widehat{A} \circ \widehat{B}) \circ (\widehat{C} \circ \widehat{D}) + (\widehat{C} \circ \widehat{D}) \circ (\widehat{A} \circ \widehat{B}) =\\
&(A \times B \times C \times D)\widehat{\ } + (B \times A \times D \times C)\widehat{\ } + (C \times D \times A \times B)\widehat{\ } + (D \times C \times B \times A)\widehat{\ }\\
&-(A \times B \times D \times C)\widehat{\ } - (B \times A \times C \times D)\widehat{\ } - (C \times D \times B \times A)\widehat{\ } - (D \times C \times A \times B)\widehat{\ }.
\end{aligned}$$

By means of $\odot$, we define a four-fold Grassmann product $\triangle(\widehat{A},\widehat{B},\widehat{C},\widehat{D})$ as follows:

Definition 6 (Four-Fold Grassmann Product). *$\triangle(\widehat{A},\widehat{B},\widehat{C},\widehat{D})$ is a 25-valued random variable defined by*

$$\begin{aligned}
&\triangle(\widehat{A},\widehat{B},\widehat{C},\widehat{D}) :=\\
&\odot(\widehat{A},\widehat{B},\widehat{C},\widehat{D}) + \odot(\widehat{A},\widehat{C},\widehat{D},\widehat{B}) + \odot(\widehat{A},\widehat{D},\widehat{B},\widehat{C}) =\\
&(A \times B \times C \times D)\widehat{\ } + (B \times A \times D \times C)\widehat{\ } + (C \times D \times A \times B)\widehat{\ } + (D \times C \times B \times A)\widehat{\ }\\
&-(A \times B \times D \times C)\widehat{\ } - (B \times A \times C \times D)\widehat{\ } - (C \times D \times B \times A)\widehat{\ } - (D \times C \times A \times B)\widehat{\ }\\
&+(A \times C \times D \times B)\widehat{\ } + (C \times A \times B \times D)\widehat{\ } + (D \times B \times A \times C)\widehat{\ } + (B \times D \times C \times A)\widehat{\ }\\
&-(A \times C \times B \times D)\widehat{\ } - (C \times A \times D \times B)\widehat{\ } - (D \times B \times C \times A)\widehat{\ } - (B \times D \times A \times C)\widehat{\ }\\
&+(A \times D \times B \times C)\widehat{\ } + (D \times A \times C \times B)\widehat{\ } + (B \times C \times A \times D)\widehat{\ } + (C \times B \times D \times A)\widehat{\ }\\
&-(A \times D \times C \times B)\widehat{\ } - (D \times A \times B \times C)\widehat{\ } - (B \times C \times D \times A)\widehat{\ } - (C \times B \times A \times D)\widehat{\ }.
\end{aligned}$$

2.2 Preference Space and Preference Space Assignment

We define preference space and preference space assignment as follows:

Definition 7 (Preference Space and Preference Space Assignment). *$\preceq_w$ is a weak preference relation on $\mathcal{F}^2$. $A \preceq_w B$ is interpreted to mean that the agent does not prefer A to B at a time in w. $\sim_w$ and $\prec_w$ are defined as follows:*

- $A \sim_w B := A \preceq_w B$ and $B \preceq_w A$,
- $A \prec_w B := A \preceq_w B$ and $A \not\succeq_w B$.

*For any $w \in$ **W**, (**W**,$\mathcal{F}$,$\preceq_w$,$\widehat{\ }$,$\times$,$+$,$-$) is called a preference space. Let **PS** denote the set of all preference spaces. ρ : **W** $\rightarrow$ **PS** is called a preference space assignment.*

2.3 Conditions for Representation

We can state necessary and sufficient conditions for representation as follows:

1. $A \preceq_w B$ or $B \preceq_w A$ (*Connectedness*),
2. If ($A_i \preceq_w B_i$ and $C_i \preceq_w D_i$ for any $i < n$),
   then (if $A_n \preceq_w B_n$, then $D_n \preceq_w C_n$),
   where $\sum_{i \leq n} \odot(\widehat{A_i}, \widehat{B_i}, \widehat{C_i}, \widehat{D_i}) = \triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n})$ (*Projectivity*)[6]

2.4 Domotor's Representation Theorem

We can prove Domotor's representation theorem as follows:[7]

Theorem 1 (Representation). *For any* $w \in \mathbf{W}$, $(\mathbf{W}, \mathcal{F}, \preceq_w, \widehat{\ }, \times, +, -)$ *satisfies (connectedness) and (projectivity) iff there are* $P_w : \mathcal{F} \to \mathbb{R}$ *and* $U_w : \mathcal{F} \backslash \emptyset \to \mathbb{R}$ *such that the following conditions hold for any* $A, B \in \mathcal{F} \backslash \emptyset$*:*

- $(\mathbf{W}, \mathcal{F}, P_w)$ *is a finitely additive probability space,*
- $A \preceq_w B$ *iff* $U_w(A) \leq U_w(B)$,
- *If* $A \cap B = \emptyset$, $U_w(A \cup B) = P_w(A|A \cup B)U_w(A) + P_w(B|A \cup B)U_w(B)$,
- *When* $A \in \mathcal{F}$, *if* $P_w(A) = 0$, *then* $A = \emptyset$.

Proof. Except that the proof is relative to world, it is similar to that of [[5]:184–194].

2.5 Scott's Separation Theorem

Domotor's representation theorem follows from Scott's separation theorem.

Theorem 2 (Separation, Scott [24]). *Let* I *be a finite-dimensional real linear vector space and let* $\emptyset \neq G \subset H \subset \mathrm{I}$, *where* $H = -\mathrm{H} = \{-v : v \in H\}$ *is finite and all its elements have rational coordinates with respect to a given basis. Then there exists a linear functional* $F : I \to \mathbb{R}$ *such that for any* $v \in H$

$$F(v) \geq 0 \textit{ iff } v \in G$$

iff for any $v, v_i \in H$ $(1 \leq i \leq n)$ *we have both*

$$(1) \quad v \in G \text{ or } -v \in G$$

and

$$(2) \text{ If } v_i \in G \textit{ for any } i < n, \textit{ then } -v \in G, \text{ where} \sum_{i \leq n} v_i = 0.$$

6 Generally, conjoint measurement requires the cancellation axiom as a necessary one. (Projectivity) can be regarded as a generalisation of the cancellation axiom.

7 In Theorem 1, we do not obtain the uniqueness result. But it does not matter when we provide CEUMPL with its semantics.

(1) corresponds to (connectedness) and (2) corresponds to (projectivity). Scott's separation theorem is based on the general criterion for the solvability of a finite set of homogeneous linear inequalities.

3 Conditional Expected Utility Maximiser's Preference Logic CEUMPL

3.1 Language

The language $\mathcal{L}_{\mathsf{CEUMPL}}$ of CEUMPL is defined as follows:

Definition 8 (Language). *Let* **S** *denote a set of sentential variables,* **WPR** *a weak preference relation symbol, and* **FCP** *a four-fold Cartesian product symbol.* $\mathcal{L}_{\mathsf{CEUMPL}}$ *is given by the following rule:*

$$\varphi ::= s \mid \top \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \mathbf{WPR}(\varphi_1, \varphi_2) \mid \mathbf{FCP}(\varphi_1, \varphi_2, \varphi_3, \varphi_4),$$

where $s \in \mathbf{S}$*, and nestings of* **FCP** *do not occur.* $\bot$*,*$\vee$*,*$\rightarrow$ *and* $\leftrightarrow$ *are introduced by the standard definitions.* **IND** *and* **SPR** *are defined as follows:*

- $\mathbf{IND}(\varphi_1, \varphi_2) := \mathbf{WPR}(\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_2, \varphi_1)$,
- $\mathbf{SPR}(\varphi_1, \varphi_2) := \mathbf{WPR}(\varphi_1, \varphi_2) \wedge \neg\mathbf{IND}(\varphi_1, \varphi_2)$.

The set of all well-formed formulae of $\mathcal{L}_{\mathsf{CEUMPL}}$ *will be denoted by* $\Phi_{\mathcal{L}_{\mathsf{CEUMPL}}}$.

3.2 Semantics

DAG In order to state $\sum_{i \leq n} \odot(\widehat{A_i}, \widehat{B_i}, \widehat{C_i}, \widehat{D_i}) = \triangle(\widehat{A_n}, \widehat{B_n}, \widehat{C_n}, \widehat{D_n})$ of (projectivity) in logical terms, we use **FCP**. To provide **FCP** with a truth definition, we use a directed acyclic graph (DAG). We got a hint about this idea from [19]. We define directedness as follows:

Definition 9 (Directedness). *A graph* G *is directed if* G *consists of a nonempty set* **W** *of vertices (possible worlds) and an irreflexive accessibility relation* R *on* **W**. G *is denoted as* $(\mathbf{W}, R)$.

We define a path as follows:

Definition 10 (Path). *A sequence* $[w_1, \ldots, w_{n+1}]$ *of vertices is a path of length* n *in* G *from* w_1 *to* w_{n+1} *if* $(w_i, w_{i+1}) \in R$ *for* $i = 1, \ldots, n$.

By means of a path, we define a cycle.

Definition 11 (Cycle). *A cycle of length* n *is a path* $[w_1, \ldots, w_n, w_1,]$ *from* w_1 *to* w_1.

By means of a circle, we define acyclicity as follows:

Definition 12 (Acyclicity). *G is acyclic if G contains no cycles.*

By means of directedness and acyclicity, we define a directed acyclic graph (DAG) as follows:

Definition 13 (DAG). *G is a directed acyclic graph (DAG) if G is both directed and acyclic.*

Remark 1. DAGs can be considered to be a generalisation of trees in which certain subtrees can be shared by different parts of the tree.

Model By means of a DAG, we define a Domotor-type structured Kripke model $\mathcal{M}$ for preference as follows:

Definition 14 (Model). *$\mathcal{M}$ is a quintuple $(\mathbf{W}, R, L, V, \rho)$, where $\mathbf{W}$ is a nonempty set of possible worlds, R is an accessibility relation on $\mathbf{W}^2$, $(\mathbf{W}, R)$ is a DAG, $L : R \to \{\pi_1, \pi_2, \pi_3, \pi_4\}$ is a function that assigns labels to the edges of the graph, any two edges leaving the same vertex have different labels, any vertex either has π_1-, π_2-, π_3- and π_4-labeled outgoing edges or none of them, V is a truth assignment to each $s \in \boldsymbol{S}$ for each $w \in \mathbf{W}$, and ρ is a preference space assignment that assigns to each $w \in \mathbf{W}$ $(\mathbf{W}, \mathcal{F}, \preceq_w, \frown, \times, +, -)$ that satisfies (connectedness) and (projectivity). For any $w_1 \in \mathbf{W}$, by $\pi_i(w_1)$ ($i =$ 1, 2, 3, 4) we mean the unique $w_2 \in \mathbf{W}$ such that $R(w_1, w_2)$ and $L(w_1, w_2) = \pi_i$ if such world exists.*

Truth Definition We can provide CEUMPL with the following truth definition:

Definition 15 (Truth). *The notion of $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMPL}}}$ being true at $w \in \mathbf{W}$ in $\mathcal{M}$, in symbols $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi$ is inductively defined as follows:*

- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} s$ *iff* $V(w)(s) = \mathbf{true}$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \top$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi_1 \wedge \varphi_2$ *iff* $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi_1$ and $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi_2$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \neg\varphi$ *iff* $(\mathcal{M}, w) \not\models_{\mathsf{CEUMPL}} \varphi$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \mathbf{FCP}(\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ *iff* $(\mathcal{M}, \pi_1(w)) \models_{\mathsf{CEUMPL}} \varphi_1$ *and* $(\mathcal{M}, \pi_2(w)) \models_{\mathsf{CEUMPL}} \varphi_2$ *and* $(\mathcal{M}, \pi_3(w)) \models_{\mathsf{CEUMPL}} \varphi_3$ *and* $(\mathcal{M}, \pi_4(w)) \models_{\mathsf{CEUMPL}} \varphi_4$,
- $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1, \varphi_2)$ *iff* $[\![\varphi_1]\!] \preceq_w [\![\varphi_2]\!]$,

where $[\![\varphi]\!] := \{w \in \mathbf{W} : (\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi\}$. *If $(\mathcal{M}, w) \models_{\mathsf{CEUMPL}} \varphi$ for all $w \in \mathbf{W}$, we write $\mathcal{M} \models_{\mathsf{CEUMPL}} \varphi$ and say that φ is valid in $\mathcal{M}$. If φ is valid in all Domotor-type structured models for preference, we write $\models_{\mathsf{CEUMPL}} \varphi$ and say that φ is valid.*

3.3 Syntax

Syntactic Counterpart of (Projectivity) We devise a syntactic counterpart of (projectivity). By developing the idea of [25], we define $\mathbf{DC}_i$ (Disjunction of Conjunctions) as follows:

Definition 16 (Disjunction of Conjunctions). For any i ($1 \leq i \leq$ 4n + 4), $\mathbf{DC}_i$ is defined as the disjunction of all the following conjunctions:

$$
\begin{aligned}
&\bigwedge_{j=1}^{n-1} d_j\mathbf{FCP}(\varphi_j, \psi_j, \chi_j, \tau_j)\\
&\wedge d_n\mathbf{FCP}(\varphi_n, \chi_n, \psi_n, \tau_n)\\
&\wedge d_{n+1}\mathbf{FCP}(\varphi_n, \tau_n, \chi_n, \psi_n)\\
&\wedge \bigwedge_{j=n+2}^{2n} d_j\mathbf{FCP}(\psi_{j-n-1}, \varphi_{j-n-1}, \tau_{j-n-1}, \chi_{j-n-1})\\
&\wedge d_{2n+1}\mathbf{FCP}(\chi_n, \varphi_n, \tau_n, \psi_n)\\
&\wedge d_{2n+2}\mathbf{FCP}(\tau_n, \varphi_n, \psi_n, \chi_n)\\
&\wedge \bigwedge_{j=2n+3}^{3n+1} d_j\mathbf{FCP}(\chi_{j-2n-2}, \tau_{j-2n-2}, \varphi_{j-2n-2}, \psi_{j-2n-2})\\
&\wedge d_{3n+2}\mathbf{FCP}(\tau_n, \psi_n, \chi_n, \varphi_n)\\
&\wedge d_{3n+3}\mathbf{FCP}(\psi_n, \chi_n, \tau_n, \varphi_n)\\
&\wedge \bigwedge_{j=3n+4}^{4n+2} d_j\mathbf{FCP}(\tau_{j-3n-3}, \chi_{j-3n-3}, \psi_{j-3n-3}, \varphi_{j-3n-3})\\
&\wedge d_{4n+3}\mathbf{FCP}(\psi_n, \tau_n, \varphi_n, \chi_n)\\
&\wedge d_{4n+4}\mathbf{FCP}(\chi_n, \psi_n, \varphi_n, \tau_n)\\
&\wedge \bigwedge_{j=1}^{n-1} e_j\mathbf{FCP}(\varphi_j, \psi_j, \tau_j, \chi_j)\\
&\wedge e_n\mathbf{FCP}(\varphi_n, \chi_n, \tau_n, \psi_n)\\
&\wedge e_{n+1}\mathbf{FCP}(\varphi_n, \tau_n, \psi_n, \chi_n)\\
&\wedge \bigwedge_{j=n+2}^{2n} e_j\mathbf{FCP}(\psi_{j-n-1}, \varphi_{j-n-1}, \chi_{j-n-1}, \tau_{j-n-1})\\
&\wedge e_{2n+1}\mathbf{FCP}(\chi_n, \varphi_n, \psi_n, \tau_n)\\
&\wedge e_{2n+2}\mathbf{FCP}(\tau_n, \varphi_n, \chi_n, \psi_n)\\
&\wedge \bigwedge_{j=2n+3}^{3n+1} e_j\mathbf{FCP}(\chi_{j-2n-2}, \tau_{j-2n-2}, \psi_{j-2n-2}, \varphi_{j-2n-2})\\
&\wedge e_{3n+2}\mathbf{FCP}(\tau_n, \psi_n, \varphi_n, \chi_n)\\
&\wedge e_{3n+3}\mathbf{FCP}(\psi_n, \chi_n, \varphi_n, \tau_n)\\
&\wedge \bigwedge_{j=3n+4}^{4n+2} e_j\mathbf{FCP}(\tau_{j-3n-3}, \chi_{j-3n-3}, \varphi_{j-3n-3}, \psi_{j-3n-3})\\
&\wedge e_{4n+3}\mathbf{FCP}(\psi_n, \tau_n, \chi_n, \varphi_n)\\
&\wedge e_{4n+4}\mathbf{FCP}(\chi_n, \psi_n, \tau_n, \varphi_n)
\end{aligned}
$$

such that exactly i of the d_j*'s and i of the* e_j*'s are the negation symbols, the rest of them being the empty string of symbols.*

By means of $\mathbf{DC}_i$, we define **DDC** as follows:

Definition 17 (Disjunction of Disjunctions of Conjunctions).

$$\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i) := \bigvee_{i=1}^{4n+4} \mathbf{DC}_i.$$

Proof System We provide CEUMPL with the following proof system.

Definition 18 (Proof System).

• *Axioms of* CEUMPL

(*A1*) *All tautologies of classical sentential logic,*

(*A2*) $\mathbf{WPR}(\varphi_1, \varphi_2) \vee \mathbf{WPR}(\varphi_2, \varphi_1)$
(***Syntactic Counterpart of Connectedness***),

(*A3*) $\mathbf{DDC}^n_{i=1}(\varphi_i, \psi_i, \chi_i, \tau_i) \to$
$(\wedge^n_{i=1}(\mathbf{WPR}(\varphi_i, \psi_i) \wedge \mathbf{WPR}(\chi_i, \tau_i)) \to (\mathbf{WPR}(\varphi_n, \psi_n) \to \mathbf{WPR}(\tau_n, \chi_n)))$
(***Syntactic Counterpart of Projectivity***),

(*A4*) $\mathbf{FCP}(\top,\top,\top,\top)$ (***Tautology and Four-Fold Cartesian Product***),

(*A5*) $\mathbf{FCP}(\varphi_1 \wedge \varphi_2, \psi_1 \wedge \psi_2, \chi_1 \wedge \chi_2, \tau_1 \wedge \tau_2)$
$\to (\mathbf{FCP}(\varphi_1, \psi_1, \chi_1, \tau_1) \wedge \mathbf{FCP}(\varphi_2, \psi_2, \chi_2, \tau_2))$
(***Conjunction and Four-Fold Cartesian Product 1***),

(*A6*) $(\mathbf{FCP}(\varphi_1, \mu, \nu, \xi) \wedge \mathbf{FCP}(\varphi_2, \mu, \nu, \xi)) \to \mathbf{FCP}(\varphi_1 \wedge \varphi_2, \mu, \nu, \xi)$
(***Conjunction and Four-Fold Cartesian Product 2***),

(*A7*) $(\mathbf{FCP}(\lambda, \psi_1, \nu, \xi) \wedge \mathbf{FCP}(\lambda, \psi_2, \nu, \xi)) \to \mathbf{FCP}(\lambda, \psi_1 \wedge \psi_2, \nu, \xi)$
(***Conjunction and Four-Fold Cartesian Product 3***),

$(\mathbf{FCP}(\lambda, \mu, \chi_1, \xi) \wedge \mathbf{FCP}(\lambda, \mu, \chi_2, \xi)) \to \mathbf{FCP}(\lambda, \mu, \chi_1 \wedge \chi_2, \xi)$
(***Conjunction and Four-Fold Cartesian Product 4***),

(*A9*) $(\mathbf{FCP}(\lambda, \mu, \nu, \tau_1) \wedge \mathbf{FCP}(\lambda, \mu, \nu, \tau_2)) \to \mathbf{FCP}(\lambda, \mu, \nu, \tau_1 \wedge \tau_2)$
(***Conjunction and Four-Fold Cartesian Product 5***),

(*A10*) $\neg\mathbf{FCP}(\varphi, \psi, \chi, \tau)$
$\leftrightarrow (\mathbf{FCP}(\neg\varphi, \psi, \chi, \tau) \vee \mathbf{FCP}(\varphi, \neg\psi, \chi, \tau)$
$\vee \mathbf{FCP}(\varphi, \psi, \neg\chi, \tau) \vee \mathbf{FCP}(\varphi, \psi, \chi, \neg\tau))$
(***Negation and Four-Fold Cartesian Product***).

• ***Inference Rules of*** CEUMPL

(*R1*) $\dfrac{\varphi_1 \quad \varphi_1 \to \varphi_2}{\varphi_2}$ (***Modus Ponens***),

(*R2*) $\dfrac{\varphi_1 \to \varphi_2}{\mathbf{WPR}(\varphi_2, \varphi_1)}$ (***Weak Preference Necessitation***),

(*R3*) $\dfrac{\varphi \wedge \psi \wedge \chi \wedge \tau}{\mathbf{FCP}(\varphi, \psi, \chi, \tau)}$ (***Four-Fold Cartesian Product Necessitation***).

A proof of $\varphi \in \Phi_{\mathsf{CEUMPL}}$ *is a finite sequence of* $\mathcal{L}_{\mathsf{CEUMPL}}$*-formulae having* φ *as the last formula such that either each formula is an instance of an axiom, or it can be obtained from formulae that appear earlier in the sequence by applying an inference rule. If there is a proof of* φ*, we write* $\vdash_{\mathsf{CEUMPL}} \varphi$.

3.4 Logical Properties

(Reflexivity), (transitivity), (connectedness) and (impartiality) are all provable in CEUMPL.

Proposition 1 (Reflexivity, Transitivity, Connectedness and Impartiality).

- $\vdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi, \varphi)$ (***Reflexivity***),
- $\vdash_{\mathsf{CEUMPL}} \mathbf{WPR}((\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_2, \varphi_3)) \to \mathbf{WPR}(\varphi_1, \varphi_3)$ (***Transitivity***),
- $\vdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1, \varphi_2) \vee \mathbf{WPR}(\varphi_2, \varphi_1)$ (***Connectedness***),
- $\vdash_{\mathsf{CEUMPL}} ((\mathbf{SPR}(\varphi_2, \varphi_3) \wedge \mathbf{SPR}(\varphi_2, \varphi_4)) \vee (\mathbf{SPR}(\varphi_3, \varphi_1) \wedge \mathbf{SPR}(\varphi_4, \varphi_1)))$
$\to ((\mathbf{IND}(\varphi_1, \varphi_2) \wedge ((\varphi_1 \wedge \varphi_3) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_1 \wedge \varphi_4) \leftrightarrow (\varphi_2 \wedge \varphi_4) \leftrightarrow \bot))$
$\to (\mathbf{WPR}(\varphi_1 \vee \varphi_3, \varphi_2 \vee \varphi_3) \leftrightarrow \mathbf{WPR}(\varphi_1 \vee \varphi_4, \varphi_2 \vee \varphi_4)))$ (***Impartiality 1***),
- $\vdash_{\mathsf{CEUMPL}} ((\mathbf{SPR}(\varphi_3, \varphi_2) \wedge \mathbf{SPR}(\varphi_2, \varphi_4)) \vee (\mathbf{SPR}(\varphi_4, \varphi_2) \wedge \mathbf{SPR}(\varphi_2, \varphi_3)))$
$\to ((\mathbf{IND}(\varphi_1, \varphi_2) \wedge ((\varphi_1 \wedge \varphi_3) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_2 \wedge \varphi_4) \leftrightarrow (\varphi_1 \wedge \varphi_4) \leftrightarrow \bot))$
$\to (\mathbf{WPR}(\varphi_1 \vee \varphi_3, \varphi_2 \vee \varphi_3) \leftrightarrow \mathbf{WPR}(\varphi_2 \vee \varphi_4, \varphi_1 \vee \varphi_4)))$ (***Impartiality 2***),
- $\vdash_{\mathsf{CEUMPL}} (\neg\mathbf{IND}(\varphi_2, \varphi_3) \wedge \neg\mathbf{IND}(\varphi_2, \varphi_4))$
$\to ((\mathbf{IND}(\varphi_1, \varphi_2) \wedge ((\varphi_1 \wedge \varphi_3) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_1 \wedge \varphi_4) \leftrightarrow (\varphi_2 \wedge \varphi_4) \leftrightarrow \bot))$
$\to (\mathbf{IND}(\varphi_1 \vee \varphi_3, \varphi_2 \vee \varphi_3) \leftrightarrow \mathbf{IND}(\varphi_1 \vee \varphi_4, \varphi_2 \vee \varphi_4)))$ (***Impartiality 3***).

Neither (contraposition), (conjunctive expansion), (disjunctive distribution) nor (conjunctive distribution) is provable in CEUMPL.

Proposition 2 (Contraposition, Conjunctive Expansion, Disjunctive Distribution and Conjunctive Distribution).

- $\nvdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1, \varphi_2) \leftrightarrow \mathbf{WPR}(\neg\varphi_2, \neg\varphi_1)$ (***Contraposition***),

- $\nvdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1, \varphi_2) \leftrightarrow \mathbf{WPR}(\varphi_1 \wedge \neg\varphi_2, \varphi_2 \wedge \neg\varphi_1)$
(***Conjunctive Expansion***),

- $\nvdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1 \vee \varphi_2, \varphi_3) \to (\mathbf{WPR}(\varphi_1, \varphi_3) \vee \mathbf{WPR}(\varphi_2, \varphi_3))$
(***Disjunctive Distribution of Left Disjunction***),

- $\nvdash_{\mathsf{CEUMPL}} \mathbf{WPR}(\varphi_1, \varphi_2 \vee \varphi_3) \to (\mathbf{WPR}(\varphi_1, \varphi_2) \vee \mathbf{WPR}(\varphi_1, \varphi_3))$
(***Disjunctive Distribution of Right Disjunction***),

- $\nvdash_{\mathsf{CEUMPL}} (\mathbf{WPR}(\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_3, \varphi_2)) \to \mathbf{WPR}(\varphi_1 \vee \varphi_3, \varphi_2)$
(***Conjunctive Distribution of Left Disjunction***),

- $\nvdash_{\mathsf{CEUMPL}} (\mathbf{WPR}(\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_1, \varphi_3)) \to \mathbf{WPR}(\varphi_1, \varphi_2 \vee \varphi_3)$
(***Conjunctive Distribution of Right Disjunction***).

(Restricted contraposition), (restricted conjunctive expansion), (restricted disjunctive distribution) and (restricted conjunctive distribution) are all provable in CEUMPL.

Proposition 3 (Restricted Contraposition, Restricted Conjunctive Expansion, Restricted Disjunctive Distribution and Restricted Conjunctive Distribution).

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow \bot) \to (\mathbf{WPR}(\varphi_1, \varphi_2) \leftrightarrow \mathbf{WPR}(\neg\varphi_2, \neg\varphi_1))$
(***Restricted Contraposition***),

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow \bot) \to (\mathbf{WPR}(\varphi_1, \varphi_2) \leftrightarrow \mathbf{WPR}(\varphi_1 \wedge \neg\varphi_2, \varphi_2 \wedge \neg\varphi_1))$
(***Restricted Conjunctive Expansion***),

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_3 \wedge \varphi_1) \leftrightarrow \bot)$
$\to (\mathbf{WPR}(\varphi_1 \vee \varphi_2, \varphi_3) \to (\mathbf{WPR}(\varphi_1, \varphi_3) \vee \mathbf{WPR}(\varphi_2, \varphi_3)))$
(***Restricted Disjunctive Distribution of Left Disjunction***),

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_3 \wedge \varphi_1) \leftrightarrow \bot)$
$\to (\mathbf{WPR}(\varphi_1, \varphi_2 \vee \varphi_3) \to (\mathbf{WPR}(\varphi_1, \varphi_2) \vee \mathbf{WPR}(\varphi_1, \varphi_3)))$
(***Restricted Disjunctive Distribution of Right Disjunction***),

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_3 \wedge \varphi_1) \leftrightarrow \bot)$
$\to ((\mathbf{WPR}(\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_3, \varphi_2)) \to \mathbf{WPR}(\varphi_1 \vee \varphi_3, \varphi_2))$
(***Restricted Conjunctive Distribution of Left Disjunction***),

- $\vdash_{\mathsf{CEUMPL}} ((\varphi_1 \wedge \varphi_2) \leftrightarrow (\varphi_2 \wedge \varphi_3) \leftrightarrow (\varphi_3 \wedge \varphi_1) \leftrightarrow \bot)$
$\rightarrow ((\mathbf{WPR}(\varphi_1, \varphi_2) \wedge \mathbf{WPR}(\varphi_1, \varphi_3)) \rightarrow \mathbf{WPR}(\varphi_1, \varphi_2 \vee \varphi_3))$
(***Restricted Conjunctive Distribution of Right Disjunction***).

3.5 Metalogic

We can prove the soundness of CEUMPL.

Theorem 3 (Soundness). *For every* $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMPL}}}$, *if* $\vdash_{\mathsf{CEUMPL}} \varphi$, *then* $\models_{\mathsf{CEUMPL}} \varphi$.

Proof. The nontrivial part of the proof is to show that (A3) is true in every model.

We can prove the completeness of CEUMPL.

Theorem 4 (Completeness). *For every* $\varphi \in \Phi_{\mathcal{L}_{\mathsf{CEUMPL}}}$, *if* $\models_{\mathsf{CEUMPL}} \varphi$, *then* $\vdash_{\mathsf{CEUMPL}} \varphi$

Proof. By Lindenbaum Lemma and Truth Lemma.

We can prove the decidability of CEUMPL.

Theorem 5 (Decidability). CEUMPL *is decidable.*

Proof. By considering the maximal number of nestings of **WPR** in a given formula.

4 Conclusions

In this paper we have proposed a new version of complete and decidable preference logic—conditional expected utility maximiser's preference logic (CEUMPL) by virtue of which we can explain an agent's preferences in terms of his beliefs and desires via conditional expected utility maximisation, which can furnish a solution to the problem Mullen posed. We have provided CEUMPL with a Domotortype semantics that is measurement-theoretic. We have provided CEUMPL with a model by developing the idea of Naumov and provided CEUMPL with a proof system by developing that of Segerberg.

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